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Allowing students to write in mathematics class can promote critical thinking, illustrate an awareness of mathematical connections, and result in clear communication as they share ideas comfortably with peers.

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As math teachers, we have often encountered students who could ace a test but not explain their reasoning. This phenomenon was disturbing to us, and we fought for years to help students both understand mathematical concepts and develop meaning for them. Since our primary goal was to develop mathematically literate students, our solution was to help them explain their thinking and focus their efforts on writing. Doing this depended on expanding tools for incorporating writing into our classroom. We worked tirelessly to help students understand that writing in mathematics should consist of—

- drawings, pictures, tables, and graphs;
- incomplete sentences that included symbols, equations, and expressions;
- explanations of problem-solving methods and justifications of processes; and
- reflections of learning.

Over time, we watched students embrace writing and become critical thinkers. They were able to organize and clearly communicate their thinking to peers and teachers. Writing aided our students in communicating their thinking in an organized and coherent manner and encouraged them to see mathematics as interconnected concepts, helping them make connections between classroom content and real-life situations (NCTM 2000).

We present four writing strategies that we used in our classrooms to promote reasoning, sense making, mathematical connections, and clear communication. We found that these strategies were an easy fit with many topics we taught. Our intentional use of these four helped students explain their understanding of the procedures they used. Each strategy can be easily adapted to various topics in mathematics.

STRATEGY 1: WORD PROBLEM ROULETTE

Barton and Heidema (2002) describe an instructional strategy, called *word problem roulette*, that challenges students to consider multiple perspectives, representations, and methods to solve problems. This strategy requires

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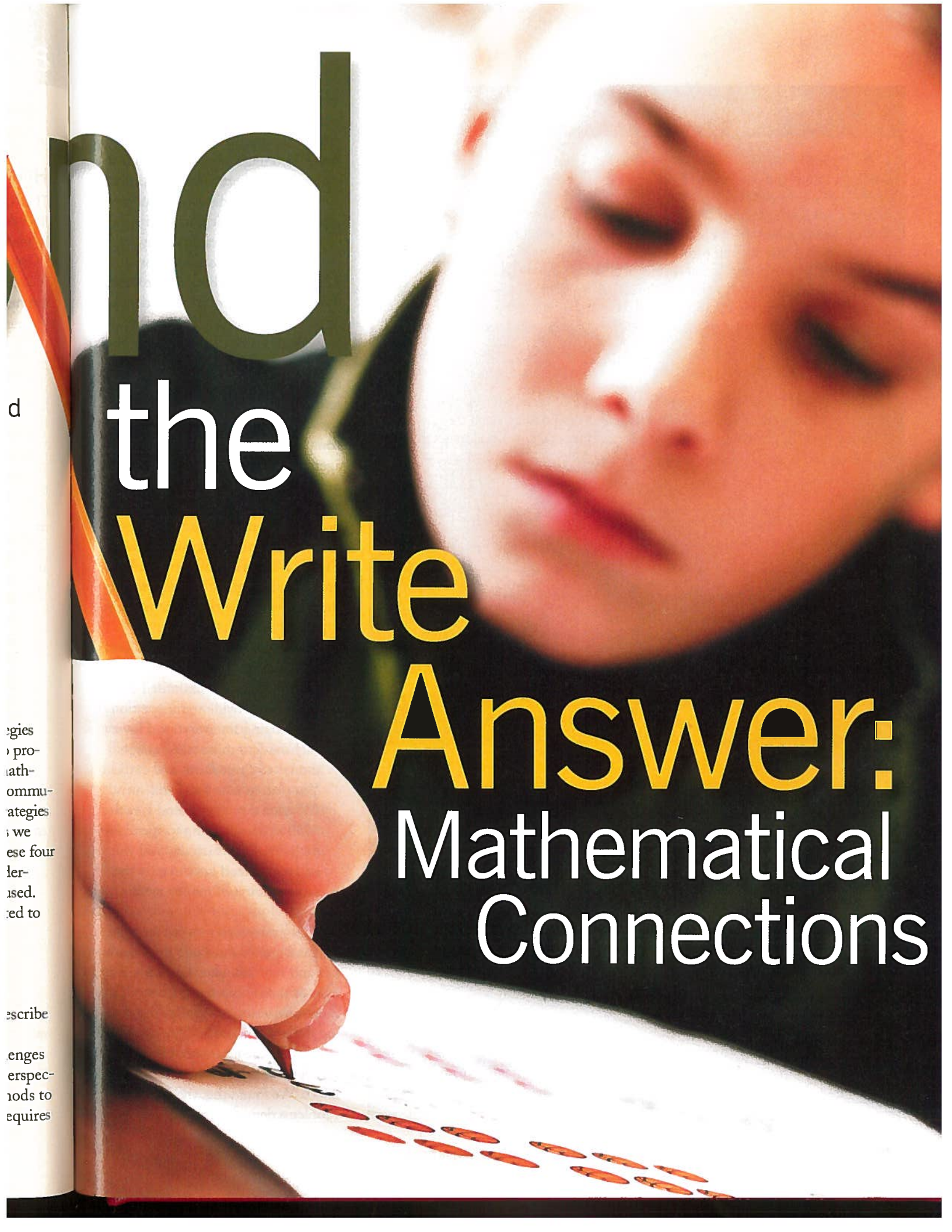
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1. Me = m; Friend = f; # of days = x
2. My equation is $m = 425 - 30x$.
3. My friend's equation is $f = 550 - 45x$.
4. Since we want to know when we'll have the same amount of money, we have to see when the equations will be equal to each other.
5. Write $425 - 30x = 550 - 45x$.
6. Add $45x$ to both sides to get $425 + 15x = 550$.
7. Subtract 425 from both sides and get $15x = 125$.
8. Divide both sides by 15 and get 8.333.
9. This means that sometime during the 8th day, we will have the same amount of money.

(a) Group A

1. Make a table showing how much money we will have after each day.
2. In my column, start with 425 and take away 30 dollars each day.
3. In my friend's column, start with 550 and take away 45 dollars each day.
4. The table should look like this:

Days	0	1	2	3	4	5	6	7	8	9
My money	425	395	365	335	305	275	245	215	185	155
My friend's money	550	505	460	415	370	325	280	235	190	145

5. On the 8th day, my friend still has more money than me.
6. On the 9th day, I have more money than my friend.
7. Sometime on the 8th day we must have the same amount of money.

(b) Group B
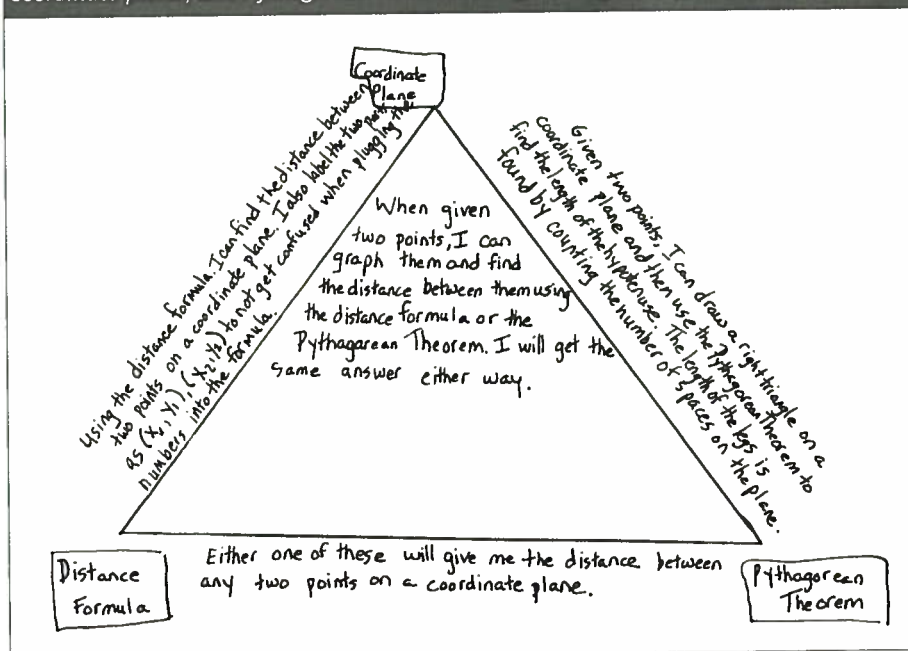
that students work collaboratively in small groups because heterogeneous groups of no more than four students encourage a variety of solution methods. Once students are placed into groups, they discuss a solution to a word problem, without writing or capturing the discussion in any way.

After reaching consensus on a single solution method, the group silently writes a group report detailing their solution steps. To begin, one individual writes the first step of their solution. The paper is then passed to another member of the group to write the second step. This paper passing continues until the solution is complete. Writing in this way encourages students to clearly communicate orally, come to a consensus, read and understand the writing of their peers, and connect their thinking to the thinking of their group members. **Figure 1** illustrates how two different groups in an algebra class approached the following History Club problem.

> The American History Club is taking a field trip to Philadelphia and Boston. The club has raised enough money to provide transportation and lodging for everyone. However, each student must take enough money to cover meals and extras. You plan to take $425 and spend $30 each day. Your friend is taking $550 and is planning to spend $45 each day. On which day will you and your friend have exactly the same amount of money remaining?

As an extension to word problem roulette, groups can share their methods and solution with the class, make a list of the various methods used, and then solve a similar word problem using different methods. This encourages students to use a variety of appropriate strategies to solve problems (NCTM 2000). We found that the Palette of Problems department (formerly called

the Menu of Problems) in this journal and the monthly calendar department in *Mathematics Teacher* were two good resources that provided a multitude of word problems that could be incorporated into our classrooms.

When initially implementing this strategy, we were confronted with students who found it difficult to express their ideas in writing and who were unable to fully understand and appreciate the thinking of their peers. Word problem roulette helped overcome these challenges by requiring that students connect their thinking to that of their classmates. Over time, students became comfortable with this strategy, were able to expand their thoughts to incorporate ideas from others, and learned to view multiple solution approaches to the problems that were presented.

STRATEGY 2: THREE-WAY TIE

An instructional strategy called three-way tie, described by Silver, Brunsting, and Walsh (2008), asks students to formulate their thinking and understanding of mathematical terms and concepts through a triangle-shaped organizational tool. From three separate but interrelated terms, such as distance formula, coordinate plane, and Pythagorean theorem, students write at least one sentence describing how a pair of words is connected along each side of the triangle. Then they write a summary describing the relationship among the three terms in the interior of the triangle (see **fig. 2**). Additional term triples to promote students' understanding of relationships among mathematical topics are listed in **figure 3**.

The three-way tie can be adapted in various ways. Once students are comfortable with the strategy, they can select their own three terms from a given unit and exchange them with classmates. This process can also be reversed. The teacher can provide the

Fig. 3 Teachers can use the following sets of terms to promote students' understanding of relationships among topics within mathematics.

- Slope-intercept form – Domain/Range – Function
- Horizontal slope – *y*-intercept – Direct variation
- Slope-intercept form – Point-slope form – Standard form
- Greatest Common Factor (GCF) – Least Common Multiple (LCM) – Prime factorization
- Median – Interquartile range (IQR) – Box-and-whisker plot
- Rational numbers – Integers – Natural numbers
- Permutation – Combination – Factorial

Fig. 4 Given descriptions in a triangular organization tool, students can determine the terms being described and provide additional information.
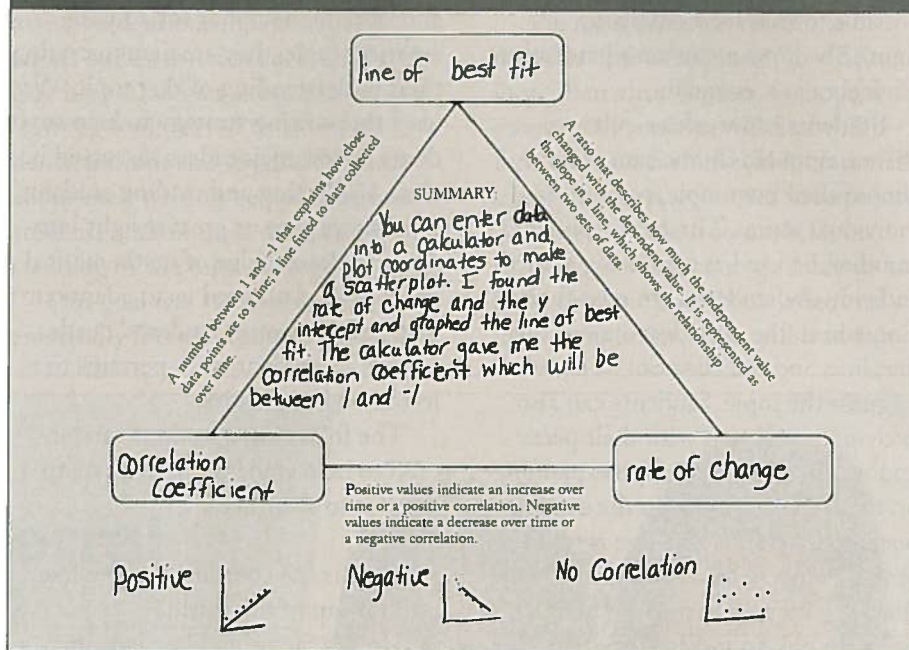
relationship among three unknown terms, and students can fill in the terms and write a summary detailing how the three are connected (see **fig. 4**). Additionally, students can present their three-way tie to the class, orally explaining and defending their thinking.

STRATEGY 3: CINQUAIN

A *cinquain* is a five-line poetic structure that promotes student writing in mathematics (Fisher and Frey 2004). Writing a cinquain can help students consider a mathematical concept in its entirety, grasp the main ideas related to that concept, and summarize this

information briefly. Through this written representation, students are directed to think about the details and critical information of mathematical concepts.

Encouraging students to write a cinquain, without the aid of textbooks or notes, can support a deeper understanding of key vocabulary and concepts. As Alvermann, Gillis, and Phelps (2012, p. 243) assert, "Much of the vocabulary learned in content areas requires *procedural knowledge*, or being able to do things with a concept, to apply it in combination with other ideas." Writing a cinquain helps students develop the procedural

Fig. 5 A *cinquain* is a five-line form poem that allows students to express what they know about a math term.

One noun (topic)
Two adjectives
Three action verbs (ending in –ing)
Four-word sentence or phrase
One synonym of topic

(a) One form of a cinquain

Algebra
complex, systematic
solving, factoring, graphing
Some answers don't exist.
math

(b) Algebra as a cinquain

knowledge needed to connect topics within mathematics and to real-life situations. **Figure 5a** displays one possible format for a cinquain; **figure 5b** shows a student's perception of algebra as a cinquain.

Students can work as a class to write a cinquain on the same topic, choose their own topic, or be given individual topics. The latter approach can then be used as a guessing game. Individually, students can read their cinquain to the class, leaving out the first line, and the class can be asked to guess the topic. Students can also exchange cinquains with their peers and write a brief paragraph expanding on the ideas presented in the other poems. Another alternative is to present students with separate elements that they must then arrange into a coherent cinquain. **Figure 6a** contains multiple elements that can challenge students to select and arrange in a coherent manner, thus allowing them to demonstrate their understanding of the mathematical concept. **Figure 6b** shows student work.

STRATEGY 4: M + M: MATH AND METAPHORS

The instructional strategy called M + M: Math and Metaphors (Silver, Brunsting, and Walsh 2008) challenges students to compare a mathematical concept to an everyday concept or object. In *Classroom Instruction That Works* (2001), Marzano, Pickering, and Pollock claim that the use of metaphors is a "highly robust activity"

that helps students make meaningful connections to their everyday lives. When students are able to create an appropriate metaphor for a mathematical topic, they are demonstrating their understanding of that topic. We used this writing strategy to help students review major ideas discussed in class. Collecting and reading students' metaphors gave us great insight into students' knowledge of mathematical concepts and allowed us to adapt our instruction to meet students' needs. **Figure 7**'s student work pertains to a lesson on inequalities.

The following questions can be used to help students connect mathematics to their lives:

- How is the coordinate plane like the map of the world?
- How is factoring like digesting food?
- How is the order of operations like a recipe for chocolate chip cookies?
- How is solving equations like skateboarding?
- How are proportions like a seesaw?
- How is graphing a line like a road map?
- How is the real number system like the United States of America?

There are multiple approaches and variations of this strategy. Students may be allowed to construct one portion of the metaphor ("How is a function like a _____?") or prepare their own metaphor within the constraint of a lesson or a unit.

Fig. 6 Using this collection of terms involving linear equations, students can express their understanding of the various connections.

Linear
Running
Start at the y-axis
Graphing
$y = mx + b$
Ordered pairs
Plotting
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Solve for unknown variables
x-intercept
y-intercept
Constant
y on one side
Rising
$(y - y_1) = m(x - x_1)$
Visual
Slope intercept
Solving

(a) Elements available

Slope-intercept
linear visual
graphing rising running
y on one side
$y = mx + b$

(b) One student's cinquain

An interesting adaptation is to take objects from the home or classroom, place them in a bag, and allow each student to select one randomly. For example, when studying probability, students in groups of three selected items such as an umbrella, a key, a

rock, a miniature baseball bat, and a plastic egg from a bag. Groups then created a list of ideas relating their selected object to probability and wrote their ideas on poster boards, which were posted in the room to invoke classroom discussion.

Working collaboratively to generate ideas helped students build on and connect to their prior knowledge before writing a metaphor. **Figure 8** showcases two different group approaches to this task. To hold students accountable, they were then asked to redraw an object from the bag and write their own metaphor in paragraph form.

A NEW PERSPECTIVE ON WRITING

Initially, our students resisted writing in math class. Expanding their understanding of what constitutes writing in math helped us overcome this challenge. The more practice the students had with writing in math class, the less opposition we encountered. We used writing strategies like these at least once per week. In the beginning, we had to allow students most of the class period to complete such tasks. As students gained experience, they required less time to complete the assignments, and more attention was focused on the resulting mathematical discussions. We adapted and used each of these strategies throughout different units of study as a way to assess student understanding and adjust our instruction accordingly. Using these strategies as assessments *for* learning tools served us and the students; prompt and specific feedback about their writing helped motivate students and improve their understanding of the topics being discussed.

Our goal was to develop mathematically literate students who could effectively communicate their understanding and ideas. Using similar writing strategies frequently and consistently helped us accomplish our goal; writing helped our students clarify their thoughts and helped them build connections between mathematical content and their lives. These strategies made writing a foundation in our math classrooms and promoted an environment that encouraged sense making, connections, collaboration, and communication between students and teachers.

We recognize that there is no one-size-fits-all approach to writing in mathematics. Rather, we hope that other math teachers will find this information useful and will intentionally adapt these strategies and incorporate them in their classrooms. In doing so, writing will encourage students to build on their prior knowledge and make connections that will lead to a
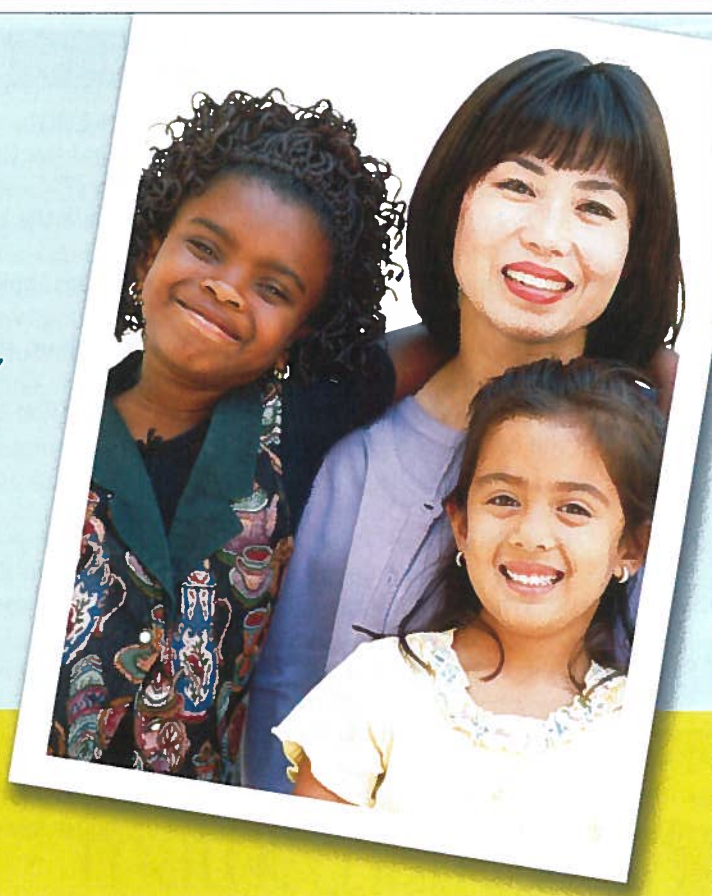
Fig. 7 This student's exploration of a metaphor involving inequalities reveals much of her understanding of the concept.

How are solving inequalities like the life of a teenager?

Today we learned how to compare real numbers like 4.09 < 4.9. We are constantly comparing ourselves to one another. For example, 1 < Tom on my math quiz. We also compare ourselves on artificial things like how we look, the type of clothes we wear, and who we hang out with. Most teenagers tend to think that clothes from American Eagle > clothes from Bue 21 or that band < football. Another thing we learned today was how to graph inequalities on a number line using open and closed circles and arrows going either left or right. We could make a number line of every student in the school as a way to compare our heights, then we would be able to see everyone ≤ me, which would be a lot. This would be graphed using a closed circle and an arrow going to the left.

Fig. 8 Two groups use the M + M writing strategy for different seed questions.

HOW IS THE CONCEPT OF PROBABILITY LIKE AN EGG?

- Easter Egg Hunt—What is the probability that I'd find the GOLD egg? What's my probability of finding more eggs than Rory?
- Egg toss—What are the chances of winning? What are the chances the farther I get from my partner?
- Breakfast—Will my mom fix scrambled eggs, fried eggs, boiled eggs, an omelet, or no eggs this week?
- Humpty Dumpty—What are the odds that I could pick up all of the pieces and put them back together again?
- Carton—How many eggs are likely to be broken if a carton of eggs were placed at the bottom of a shopping bag?

(a)

HOW IS THE CONCEPT OF PROBABILITY LIKE A KEY?

- Car key—People have a greater probability of an accident than traveling by airplane. What are the chances of my parents buying me the car of my dreams?
- House key—Someone breaking into my house
- Key to my heart—Finding true love
- Answer key—How many students in this class would cheat if we had the answer key to a test?
- Key lime pie—How many days will the cafeteria serve key lime pie this year?
- Dad's keys—How many times will my dad pick the house key without looking?

(b)

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rich and complete understanding of mathematics. As one student commented, "I like how we don't always just take notes or do bookwork. It helps me to learn in a different way."

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